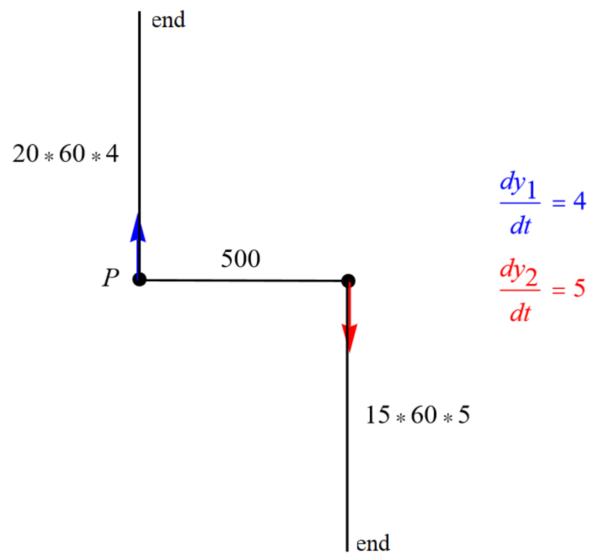
Exercise 19

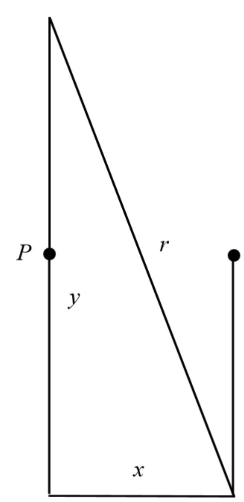
A man starts walking north at 4 ft/s from a point P. Five minutes later a woman starts walking south at 5 ft/s from a point 500 ft due east of P. At what rate are the people moving apart 15 min after the woman starts walking?

Solution

Draw a schematic of the paths after 20 minutes. Let y_1 be the distance the man walks, and let y_2 be the distance the woman walks. Multiply the walking speeds by the respective times spent walking ($20 * 60$ seconds for the man and $15 * 60$ seconds for the woman) to obtain the distances the man and woman walk.



At any time, the distance between the man and woman r is related to the horizontal distance x and the vertical distance y by the Pythagorean theorem.



$$r^2 = x^2 + y^2$$

The derivative of r with respect to t, dr/dt, represents the rate at which the man and woman are moving apart. Differentiate both sides of the Pythagorean theorem with respect to t and use the chain rule.

$$\frac{d}{dt}(r^2) = \frac{d}{dt}(x^2 + y^2)$$

$$2r \cdot \frac{dr}{dr} = 2x \cdot \frac{dx}{dt} + 2y \cdot \frac{dy}{dt}$$

$$r\frac{dr}{dt} = x\frac{dx}{dt} + y\frac{dy}{dt}$$

$$\frac{dr}{dt} = \frac{x\frac{dx}{dt} + y\frac{dy}{dt}}{r}$$

$$= \frac{x\frac{dx}{dt} + y\frac{dy}{dt}}{\sqrt{x^2 + y^2}}$$

Note that y is the sum of the man and woman's distances: $y = y_1 + y_2$.

$$\frac{dr}{dt} = \frac{x\frac{dx}{dt} + (y_1 + y_2)\frac{d}{dt}(y_1 + y_2)}{\sqrt{x^2 + (y_1 + y_2)^2}}$$

$$= \frac{x\frac{dx}{dt} + (y_1 + y_2)\left(\frac{dy_1}{dt} + \frac{dy_2}{dt}\right)}{\sqrt{x^2 + (y_1 + y_2)^2}}$$

$$= \frac{x\frac{dx}{dt} + (y_1 + y_2)(4 + 5)}{\sqrt{x^2 + (y_1 + y_2)^2}}$$

$$= \frac{x\frac{dx}{dt} + 9(y_1 + y_2)}{\sqrt{x^2 + (y_1 + y_2)^2}}$$

The horizontal distance between the man and woman does not change as time passes, so $dx/dt = 0$.

$$\frac{dr}{dt} = \frac{x(0) + 9(y_1 + y_2)}{\sqrt{x^2 + (y_1 + y_2)^2}}$$

$$= \frac{9(y_1 + y_2)}{\sqrt{x^2 + (y_1 + y_2)^2}}$$

Evaluate dr/dt at $t = 20$ to get the desired rate 15 min after the woman starts walking.

$$\left.\frac{dr}{dt}\right|_{t=20} = \frac{9[(20 * 60 * 4) + (15 * 60 * 5)]}{\sqrt{(500)^2 + [(20 * 60 * 4) + (15 * 60 * 5)]^2}} = \frac{837}{\sqrt{8674}} \approx 8.98702\ \frac{\text{ft}}{\text{s}}$$